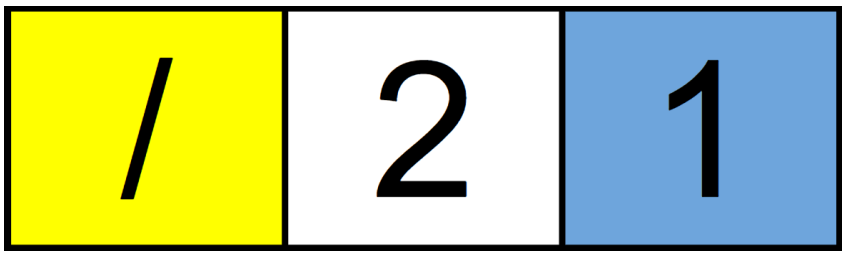

Divide to One (divide21 or /21)

A Strategic Game of Digit Manipulation and Divisibility

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Abstract

Divide21 is a mathematical game designed to teach divisibility, prime numbers, and strategic reasoning through digit manipulation and one-digit division. Players alternate turns modifying digits and attempting divisions to reach a score of 21 or reduce the number to 1. This paper introduces the game mechanics, explores its strategic complexity, and raises open questions about optimal play and computational analysis.

Introduction

This paper introduces a new mathematical game called divide21 (or Divide to One), which blends number theory, game theory, and algorithm decision making.

Inspired by my enormous passion for mathematics, computer science, and strategy games, divide21 was created to help learners, especially students in foundational math courses, to understand and master core concepts such as divisibility, prime numbers, factors and multiples, through a fun and interactive play. The game also cultivates mental math, logical reasoning and strategic thinking, making it both educational and intellectually stimulating.

This game is intended to present an engaging challenge that fosters curiosity, deeper exploration and research.

Understanding divisibility, prime numbers, and factorization is foundational in mathematics, yet these concepts are often taught in abstract or procedural ways that can feel disconnected from real world reasoning. Divide21 transforms these ideas into an interactive experience, allowing learners to engage with number theory through strategic play. By embedding mathematical logic into a game format, it encourages intuitive exploration, pattern recognition, and decision making skills that are essential not only in mathematics but also in computer science, cryptography, and algorithm design.

Moreover, the structure of the game introduces players to the tension between short-term gains and long-term strategy, a concept central to the game theory and optimization. This makes divide21 not just a learning tool, but a platform for deeper mathematical inquiry and AI experimentation.

This paper presents the rules and mechanics of divide21, gives illustrative examples of gameplay, showing strategic choices and outcomes, it also provides an analysis of the complexity of the game, some open questions related to the optimal play, game termination, and computational difficulty are also presented, and future directions for Artificial Intelligence (AI) and Machine Learning (ML) development using reinforcement learning to improve gameplay.

Game Rules

Initial Setup

A random positive integer **X** is generated, such that:

- It has more than one digit, which is determined before the game starts.
- All of its divisors (except 1) must also have more than one digit.

Player Actions

Players alternate turns, and on each turn may:

- Change a single digit of **X**, such that:
    - **X** does not become 0 or 1
    - All ten digits (0-9) must be used at a given index before any of them is reused.
    - Leading zeros are not allowed; if they appear, they cannot be changed.
- Attempt division of **X** by a one-digit number (2-9):
    - If successful, the divisor is added to the player's score.
    - If unsuccessful, the divisor is subtracted from the score.
    - If a player misses a valid division opportunity, the largest one-digit factor of **X** is subtracted from the player's score.

After Division

- The quotient becomes the new **X**, and the player may keep dividing it to get extra points.
- Digits in the quotient cannot be reused at their respective index, unless all ten one-digit numbers (2-9) have been used.
- The timer does not reset.

Win and Loss Conditions

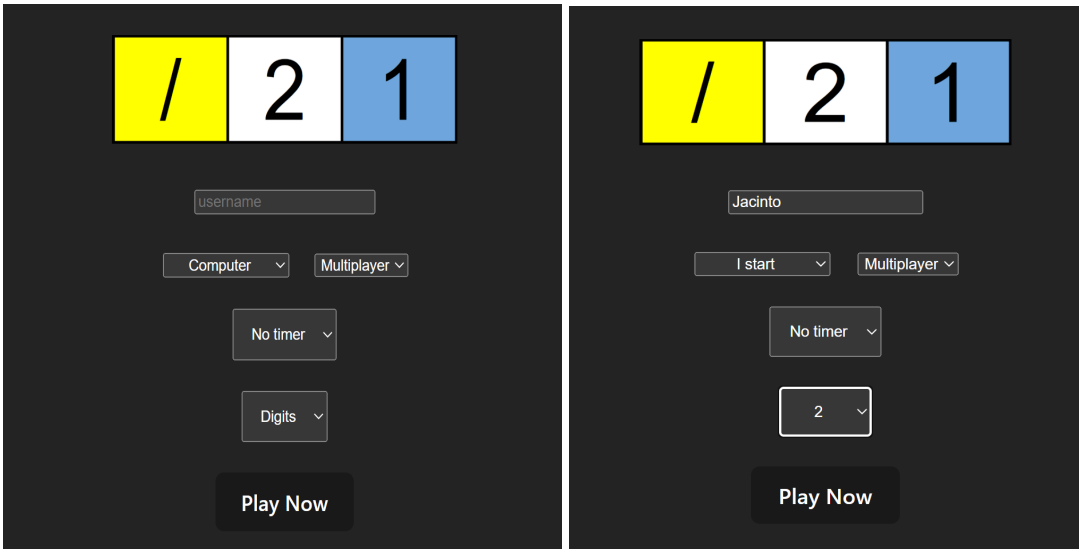
- Win: Reach a quotient 1 or accumulate 21+ points.
- Loss: Run out of time or drop to -21 points or fewer.

Examples

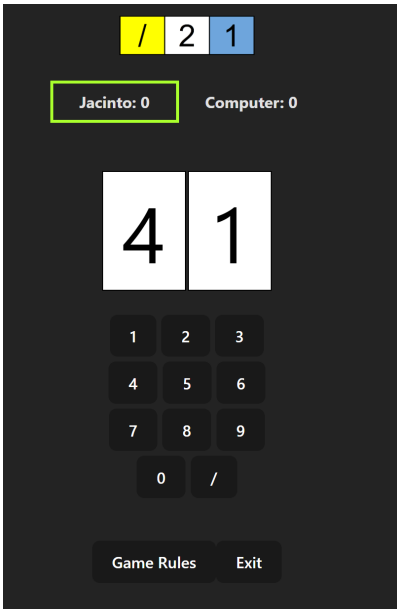
Here are a few examples of the game, with the design from the web application I have created, which are current at the time of this writing.

Example 1:

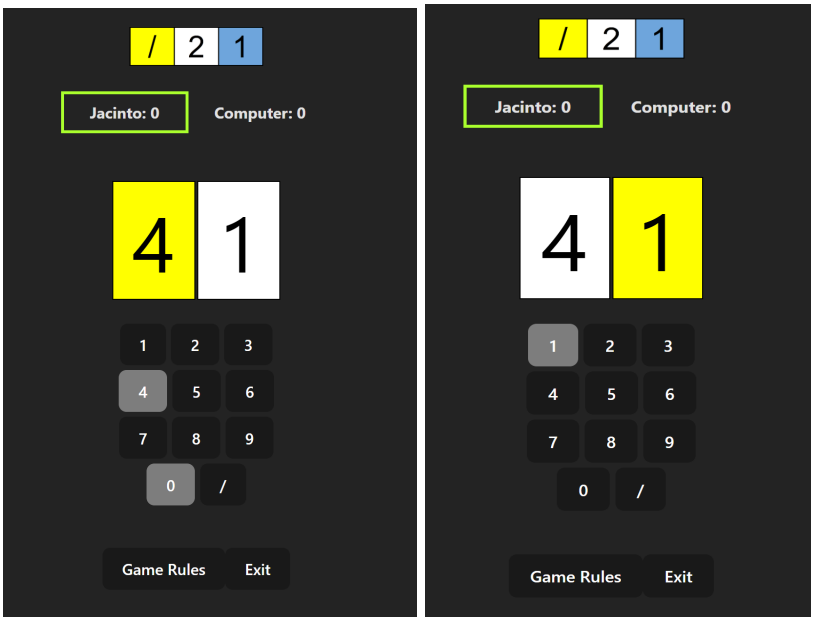
A player plays the Computer engine and chooses to start making the initial digit change and also the player determines that the initial number has two digits:



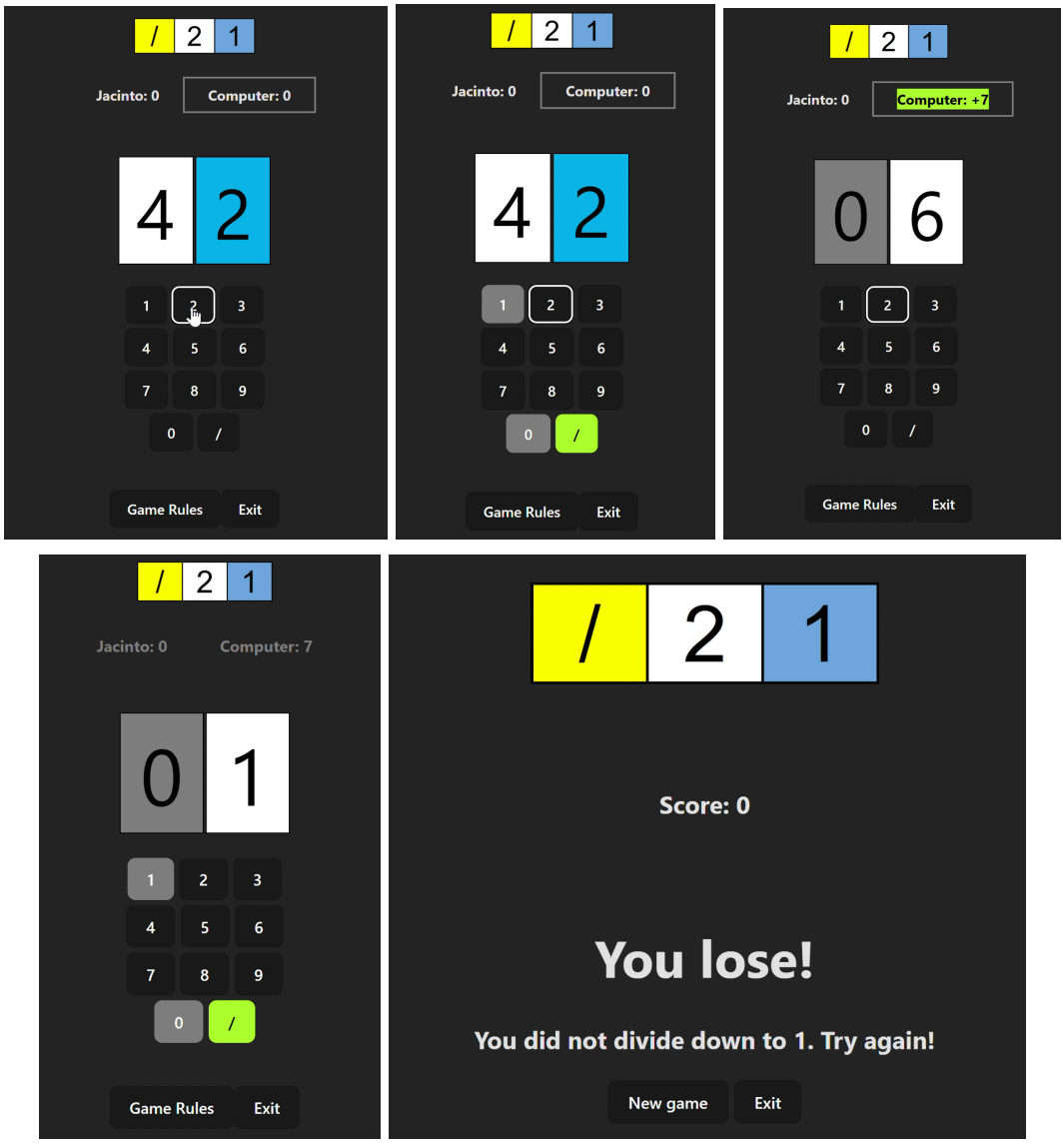
Then a two digit number with no one-digit factors is generated at random. In this case the number was 41, which is also a prime number:



When clicking on the digit 4, on the left, we see that the digit 0 and 4 are not allowed to be played. 4 can't be played because it is the original digit there, and 0 can't be played because it creates a leading zero, and additionally, it makes the number 1. Notice that when a digit is clicked, it is highlighted in yellow. Similarly, when clicking on the digit 1, on the right, we notice that 1 can't be played because it is already used at that index, as it is part of the original number.

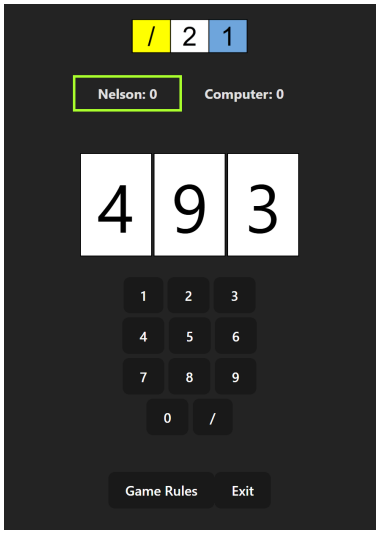


So, for the first move, the player changed the digit 1 to 2, so the number became 42. The Computer immediately figured that 42 is divisible by 7, its greatest one-digit divisor, (42/7 = 6), so it pressed the division button (“/”) and then pressed 7, so the number became 6, and the Computer earned 7 points (Computer: +7, as seen in the UI) and then it proceeded to divide it by 6 to get the number to 1 and win the game.
There are a few things to notice. First, when the Computer pressed the division button, the numbers 0 and 1 were unavailable because, we can't divide by 0 and because dividing by 1 does not change the number and hence no points would be awarded nor deducted. Second, when a digit is changed, it is highlighted as blue. Notice that when the Computer divided 42 by 7 the result was “06” with the digit “0” being unavailable for change because it is a leading zero:

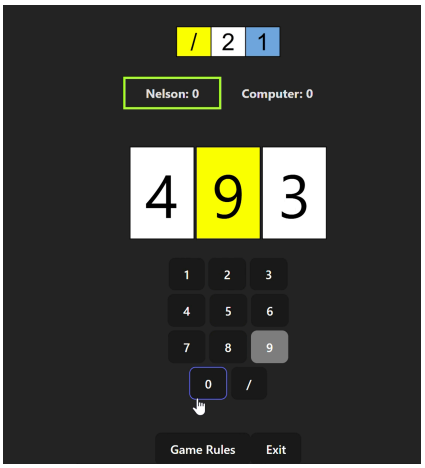


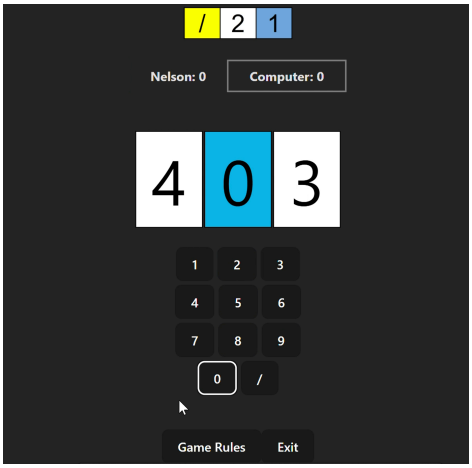
Example 2:

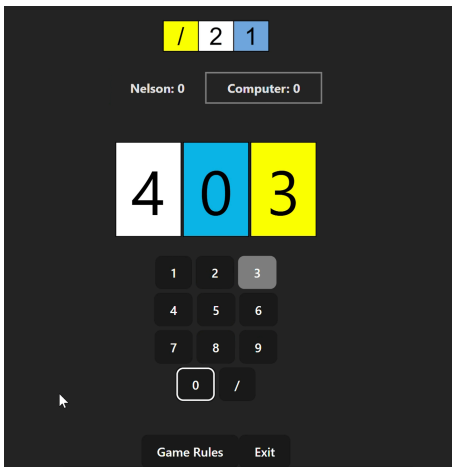
A player plays the Computer and the player chooses to start making the initial move, the number of digits is 3, and the number generated is 493.

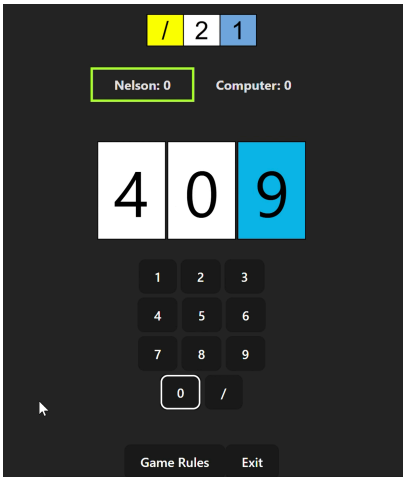


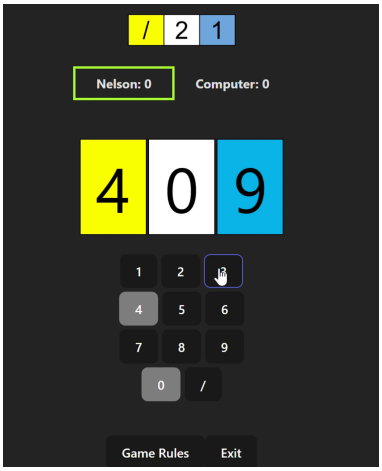
Since 493 does not have any one-digit factor (greater than 1), 493=17x29, for the first move the player changes the middle digit 9, to 0, to get 403. Then the computer changes the rightmost digit 3, to 9, so the number is 409, then the player changes the leftmost digit 4, to 3, so the number is now 309, but that is divisible by 3, which the Computer does divide by, so get the number to 103, then change the middle digit 0, to 7, to get 173, then the player changes the rightmost digit 3, to 2 to get 172, but this number is divisible by 4, so the Computer gets it to 43, and then changes the digit 4, to 1, so now the number is 13, then Nelson changes the digit 3, to 7, and the computer changes the digit 7 to 4, which was a blunder because Nelson divides 14 by 2 to get 7 and then by 7 to win the game. These are some screenshots of the game:

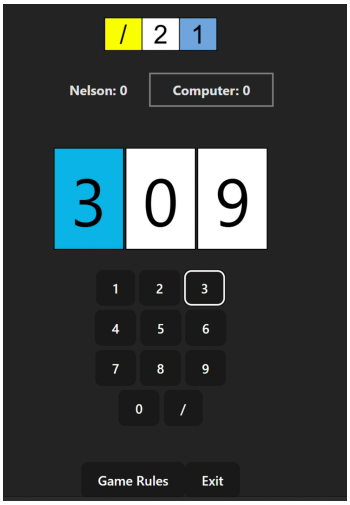

/ 2 1
Nelson: 0
Computer: 0
4 9 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

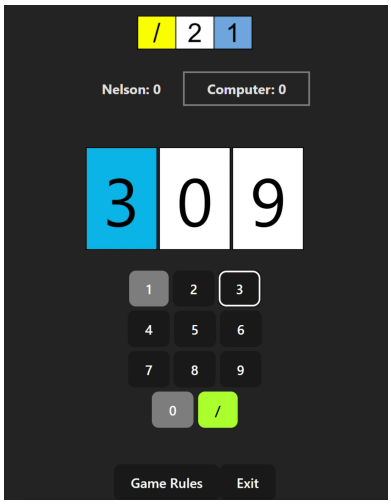

/ 2 1
Nelson: 0
Computer: 0
4 0 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

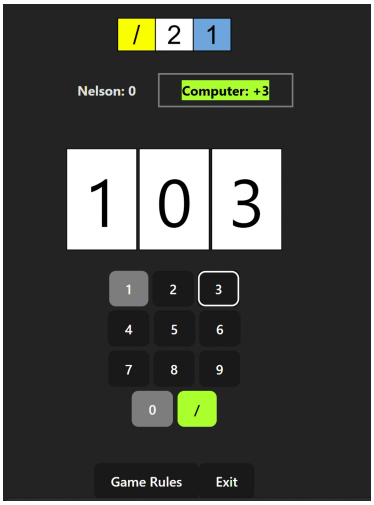

/ 2 1
Nelson: 0
Computer: 0
4 0 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

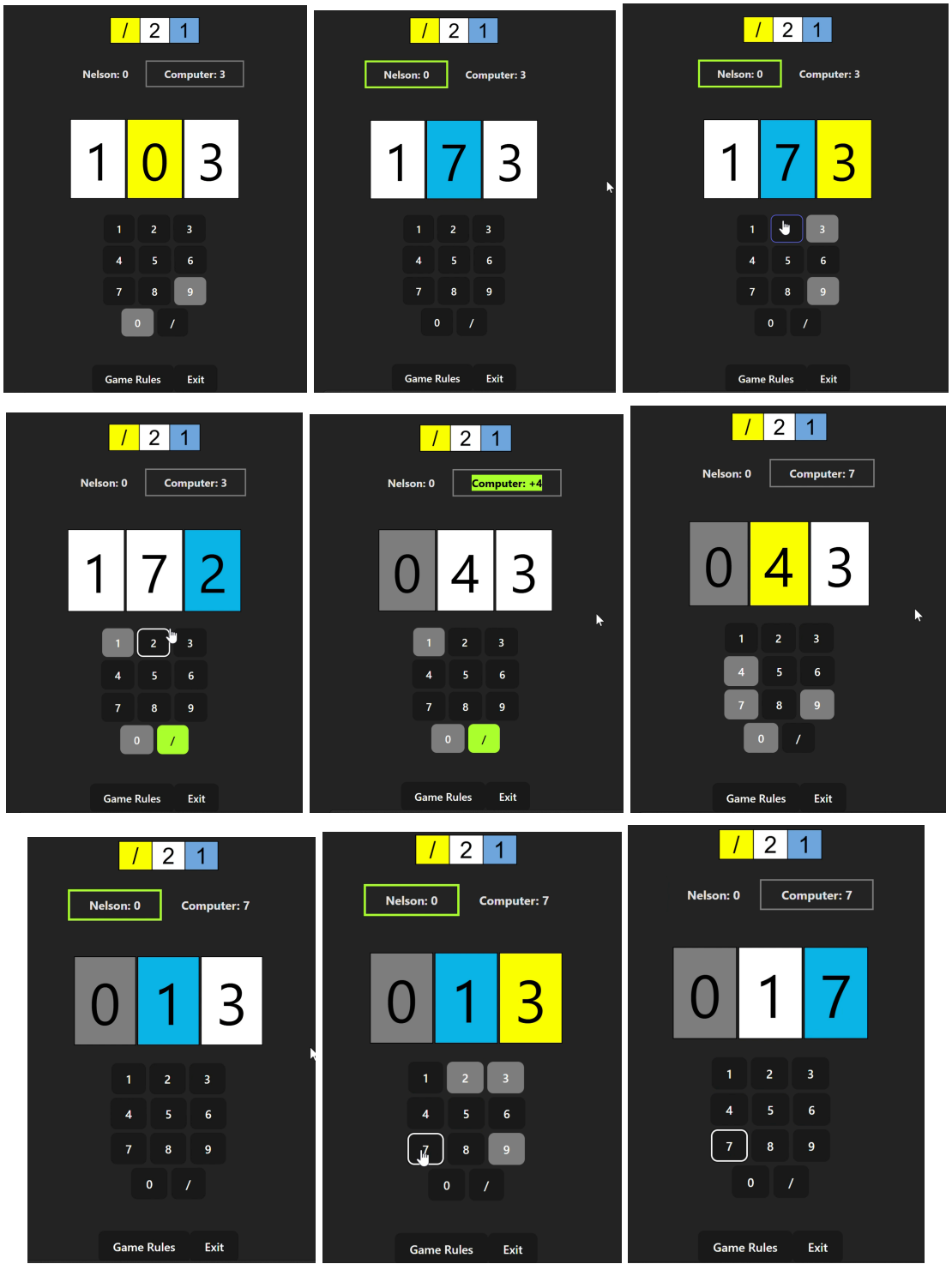

/ 2 1
Nelson: 0
Computer: 0
4 0 9
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit


/ 2 1
Nelson: 0
Computer: 0
4 0 9
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

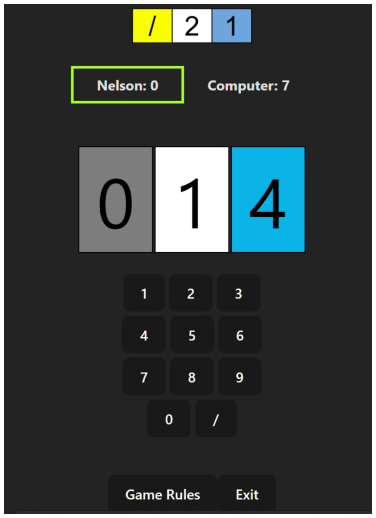

/ 2 1
Nelson: 0
Computer: 0
3 0 9
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

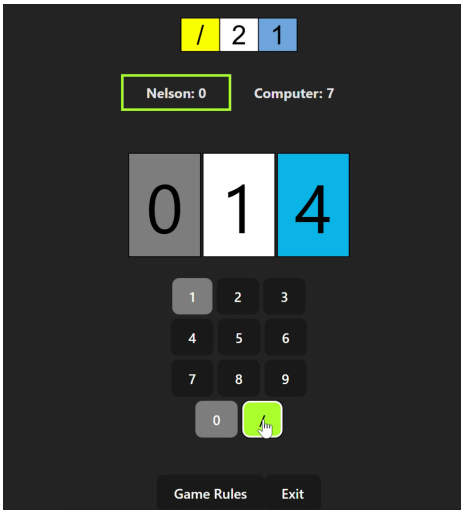

/ 2 1
Nelson: 0
Computer: 0
3 0 9
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

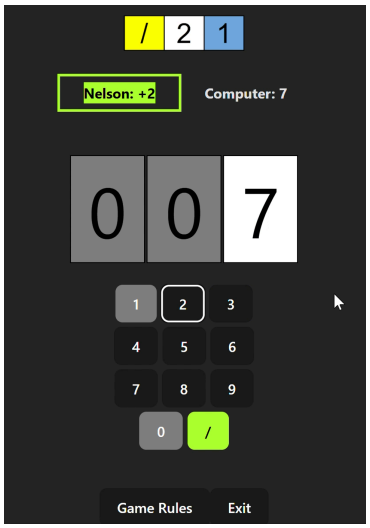

/ 2 1
Nelson: 0
Computer: +3
1 0 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

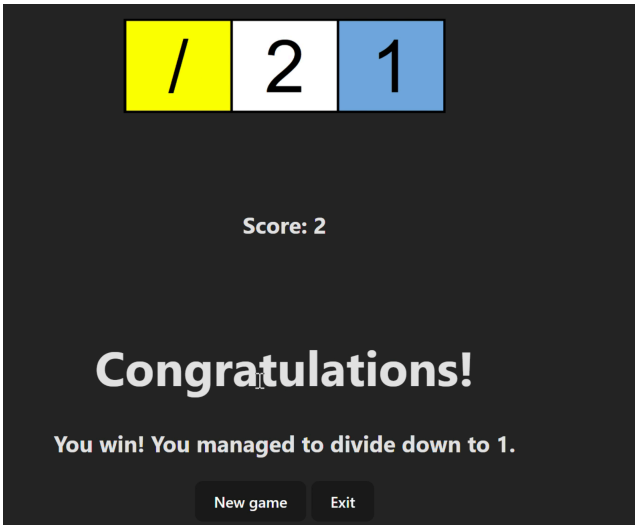

/ 2 1
Nelson: 0
Computer: 3
1 0 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 3
1 7 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 3
1 7 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 3
1 7 2
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: +4
0 4 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 7
0 4 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 7
0 1 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 7
0 1 3
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit
/ 2 1
Nelson: 0
Computer: 7
0 1 7
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit

/ 2 1
Nelson: 0
Computer: 7
0 1 7
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit


/ 2 1
Nelson: 0
Computer: 7
0 1 4
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit


/ 2 1
Nelson: 0
Computer: 7
0 1 4
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit


/ 2 1
Nelson: +2
Computer: 7
0 0 7
1 2 3
4 5 6
7 8 9
0 /
Game Rules
Exit


/ 2 1
Score: 2
Congratulations!
You win! You managed to divide down to 1.
New game
Exit

Play Online

I have developed a working version of divide21, available at: www.divide21.com

Strategy and Complexity

Divide21 has a rich strategic space and several levels of complexity because of its high branching factor.

Each state of the game is defined by, the current value of **X,** tracking list of used digits (and forbidden digits, such as 0 as the leftmost digit) at each index, the players' scores, the valid one-digit divisors of **X** and turn order (who goes first, who goes next, and so on). In addition, the states of the game grows very rapidly as the initial number of digits of **X** increases. That also happens with the branching factor of digit changes (up to 9 per digit, excluding the ones already used), and the potential multiple chained divisions.

At each turn, the player must choose which digit to modify and what new digit to use under the digit-history constraints. Furthermore, once **X** becomes divisible by a one-digit number (2-9), the player must then decide whether to divide it immediately and earn points (and maybe even start a chain of divisions that lead to 1) or miss that division intentionally and instead make a digit change, which causes point deduction, but gives a greater advantage in the future. This is a key tension in divide21: greedy scoring vs long-term positioning. Dividing by the largest one-digit factor may result in the most points in the short term, however, it might lead to a new number with fewer divisor options or unlock easy paths for the opponent.

Current Development

The current Computer engine does not play optimally. For example, changing 17 to 14 enabled the opponent to win the game. I am actively examining ways to build an AI using reinforcement learning to improve gameplay.

I have also been expanding the web application (divide21.com) to attract users and generate data for academic research in: Algorithm Design, Discrete mathematics, Game Theory, Number Theory, AI/ML and Reinforcement Learning.

Open questions

1. Is dividing **X** (the initial number) by its largest possible one-digit divisor (factor) in order to maximize points per division always the optimal strategy to win the game?
2. How does the initial value of **X** influence the outcome of the game? How does it (or does not) give an advantage to the player with the first turn?
3. Does the initial value of **X** ever forces the player with the first turn to make a digit change that will make **X:**
    a. A number with a one digit factor, thus giving the following player the opportunity to get points?
    b. A number with a one digit factor that immediately leads to a series of divisions until the quotient is 1 or the player has 21+ points?
4. If all players always play optimally:
    a. Will the games always be finite? Can loops occur?
    b. Is it possible to always win the game only by one method? (getting the quotient to 1 or getting 21+ points)?
    c. What's the maximum number of primes that **X** can be during the game? Can we list them in advance before the first move is made?
5. For a given number of digits in **X:**
    a. How many possible games are there?
    b. What are the longest and shortest possible games?
    c. What is the computational complexity of finding an optimal path?
    d. What is the maximum number of players in the game so that it is guaranteed that everyone gets at least one turn, assuming optimal play?

Conclusion

Divide21 is a unique mathematical gameplay, combining divisibility puzzles, digit manipulation, and strategic decision making. It offers a platform for both gameplay and deeper mathematical exploration, with potential applications in education, AI research, and discrete mathematics.

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